

SOLUTION	MARKS	REMARKS
1. (a) $x^2 - 2x + 1 = (x - 1)^2$	2A	or $(x-1)(x-1)$
(b) $x^2 - 2x + 1 - 4y^2 = (x - 1)^2 - 4y^2$ $= (x - 1 - 2y)(x - 1 + 2y) \dots$ $= (x - 2y - 1)(x + 2y - 1)$	1M 1M+1A 5	for $()^2 - 4y^2$ 1M for diff. of 2 sq's. No marks for $x^2 - 4y^2 = (x-2y)(x+2y)$
2. Let $f(x) = 2x^3 + ax^2 + bx - 2$ Putting $x = 2$, $f(2) = 4a + 2b + 14$ As $x - 2$ divides $f(x)$, $4a + 2b + 14 = 0$. Similarly $f(-1) = a - b - 4$ $= 0$ Solving the equations, $6a + 6 = 0$ $a = -1, b = -5$	1A 1M 1A 1A 1A+1A 5	for $f(2) = 0$ or $f(-1) = 0$
(Syll A)		
3. (a) $\sqrt{\frac{3^{5k+2}}{27^k}} = \sqrt{\frac{3^{5k+2}}{(3^3)^k}}$ $= 3^{k+1} \dots$	1A 1A	
(b) $\frac{\log a^3 b^2 - \log a b^2}{\log \sqrt{a}} = \frac{\log \frac{a^3 b^2}{a b^2}}{\log \sqrt{a}} \dots$ $= \frac{\log a^2}{\log \sqrt{a}}$ $= \frac{2 \log a}{\log a} \dots$ $= 4$	1A 1A 1A 1A 5	or $= \frac{\log a^3 + \log b^2 - \log a - \log b^2}{\log \sqrt{a}}$ $= \frac{3 \log a - \log a}{\log a}$ 1A
(Syll B)		
3. $3^{2x} + 3^x - 2 = 0$ $(3^x)^2 + 3^x - 2 = 0 \dots$ $(3^x - 1)(3^x + 2) = 0$ $3^x = 1 \text{ or } 3^x = -2$ (Rejecting $3^x = -2$) $x = 0$	1M 1A 1A 1A 1A 5	$(3^x)^2$) Accept $3^x = 1$)

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P.3

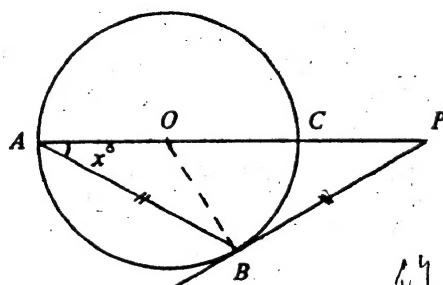
87 MATHS (SYLL A/B)

SOLUTION

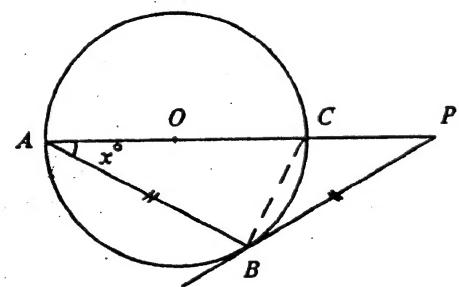
MARKS

REMARKS

7.



14 15 16



Join OB.

As OA and OB are radii of the same circle,

$$\angle OBA = \angle PAB = x^\circ \dots \dots \dots$$

Since PB is a tangent,

$$\angle OBP = 90^\circ$$

Given that BA = BP

$$\angle BPA = \angle PAB = x^\circ \dots \dots \dots$$

$$x + x + x + 90 = 180$$

$$\begin{aligned} 3x &= 90 \\ x &= 30 \end{aligned} \quad \text{得 2 分}$$

Alternatively:

1A Join BC.

As PB is a tangent,
 $\angle CBP = \angle PAB = x^\circ$.

1A Since AC is a diameter,
 $\angle ABC = 90^\circ$
etc.

1A

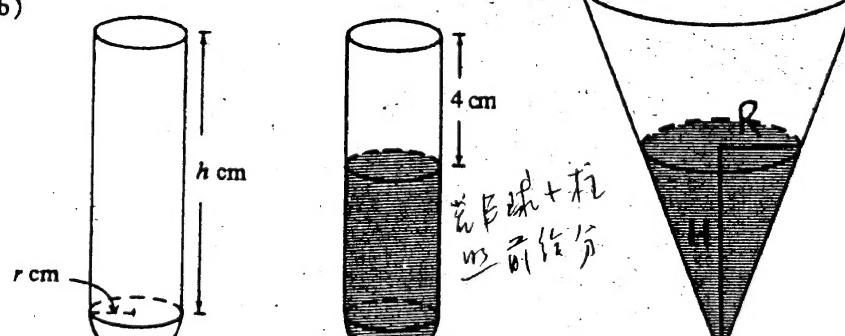
1A

1A

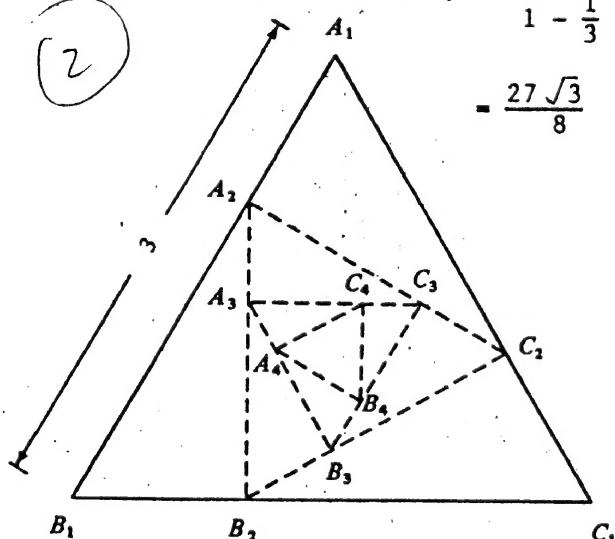
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P.5

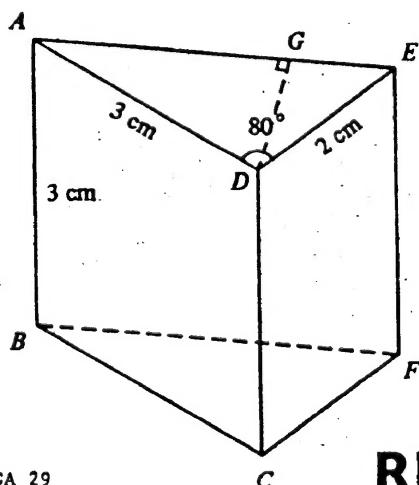
87 MATHS (SYLL A/B)

SOLUTION	MARKS	REMARKS
<p>9. (a) (i) Capacity of hemispherical part</p> <p>(3) $\frac{1}{2} \times \frac{4}{3} \pi r^3$ <small>若將寫出有 $r^3 = \frac{3}{4}V$</small> $\frac{1}{2} \times \frac{4}{3} \pi (108\pi)$ <small>則可得 $1M$</small></p> <p>$r^3 = 27$ $r = 3$</p> <p>Capacity of cylindrical part</p> <p>$\pi r^2 h$ $9\pi h$ <small>若 r 有錯，仍給此 $1M$</small></p> <p>$9\pi h = \frac{5}{6} (108\pi)$</p> <p>$h = 10$</p> <p>(ii) Volume of space = $\pi(3^2)(4)$</p> <p>(3) Volume of water = $108\pi - (\pi)(3^2)(4)$ <small>若 r 有錯 $1M$</small> $= 72\pi \text{ cm}^3$ <small>若 72π 有錯 $1A$ 不給 $\frac{1}{9}$</small></p>	<p>1M+1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M+1M</p> <p>1A</p>	<p>若寫 $r = \dots \text{ cm}$ <small>若寫錯 $1M$</small></p> <p>Alternatively: Volume $= \pi(3)^2(10-4) + \frac{108\pi}{6}$... 1M+1M $= 72\pi \text{ cm}^3$ 1A</p>
<p>(b)</p> <p>(3) </p> <p>Let radius and depth of water be R and H.</p> <p>$\frac{1}{3}\pi R^2 H = 72\pi$ <small>若上寫錯 $1M$</small></p> <p>$R^2 H = 216$</p> <p>Capacity of vessel = $\frac{1}{3}\pi(2R)^2(2H)$</p> <p>$= \frac{8}{3}\pi R^2 H$</p> <p>$= \frac{8}{3}\pi \cdot (216)$</p> <p>$= 576\pi \text{ cm}^3$</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>若 π 有錯 $1M$</p> <p>-1 if unit not given</p>
<p>Alternatively:</p> <p>Since height of vessel = $2 \times$ height of water Capacity of vessel = $2^3 \times 72\pi$ $= 576\pi \text{ cm}^3$</p>	<p>2M</p> <p>1A</p> <p>1A</p>	<p>-1 if unit not given</p>

SOLUTION	MARKS	REMARKS
10. (a) Since the triangle is equilateral, $\angle A_1 = 60^\circ$, $T_1 = \frac{1}{2} (3)(3)(\sin 60^\circ)$ $= \frac{9\sqrt{3}}{4}$	1M <u>1A</u> <u>2</u>	
(b) (i) Since $A_2B_1 = 2$, $B_1B_2 = 1$ and $\angle B_1 = 60^\circ$, $\angle B_1B_2A_2 = 90^\circ$ $\therefore A_2B_2 = \sqrt{3} \dots \dots \dots$	1M 1A	Alternatively: By cosine rule, $(A_2B_2)^2 = 2^2 + 1^2 - 2(2)(1)\cos 60^\circ$ $= 3$ $\therefore A_2B_2 = \sqrt{3}$
(ii) $\triangle A_2B_2C_2$ and $\triangle A_1B_1C_1$ are similar. The ratio of their sides is $\sqrt{3} : 3$. $\therefore T_2 = \frac{9\sqrt{3}}{4} \left(\frac{\sqrt{3}}{3} \right)^2$ $= \frac{3\sqrt{3}}{4} \quad \text{標記} \frac{\sqrt{3}}{4}$	1M <u>1A</u> <u>4</u>	
(c) (i) The common ratio = $\frac{1}{3}$ (1M, \therefore $\frac{1}{3}$ は $\frac{1}{3}$ の $\frac{1}{3}$ である) (ii) $T_n = \frac{9\sqrt{3}}{4} \left(\frac{1}{3} \right)^{n-1}$ (iii) $T_1 + T_2 + \dots + T_n = \frac{9\sqrt{3}}{4} \cdot \frac{1 - \left(\frac{1}{3} \right)^n}{1 - \frac{1}{3}}$ $= \frac{27\sqrt{3}(1 - \frac{1}{3^n})}{8}$ (iv) The sum to infinity = $\frac{\frac{9\sqrt{3}}{4}}{1 - \frac{1}{3}} = \frac{27\sqrt{3}}{8} \dots \dots \dots$	1M 1M 1M 1A	
	<u>1A</u> <u>6</u>	

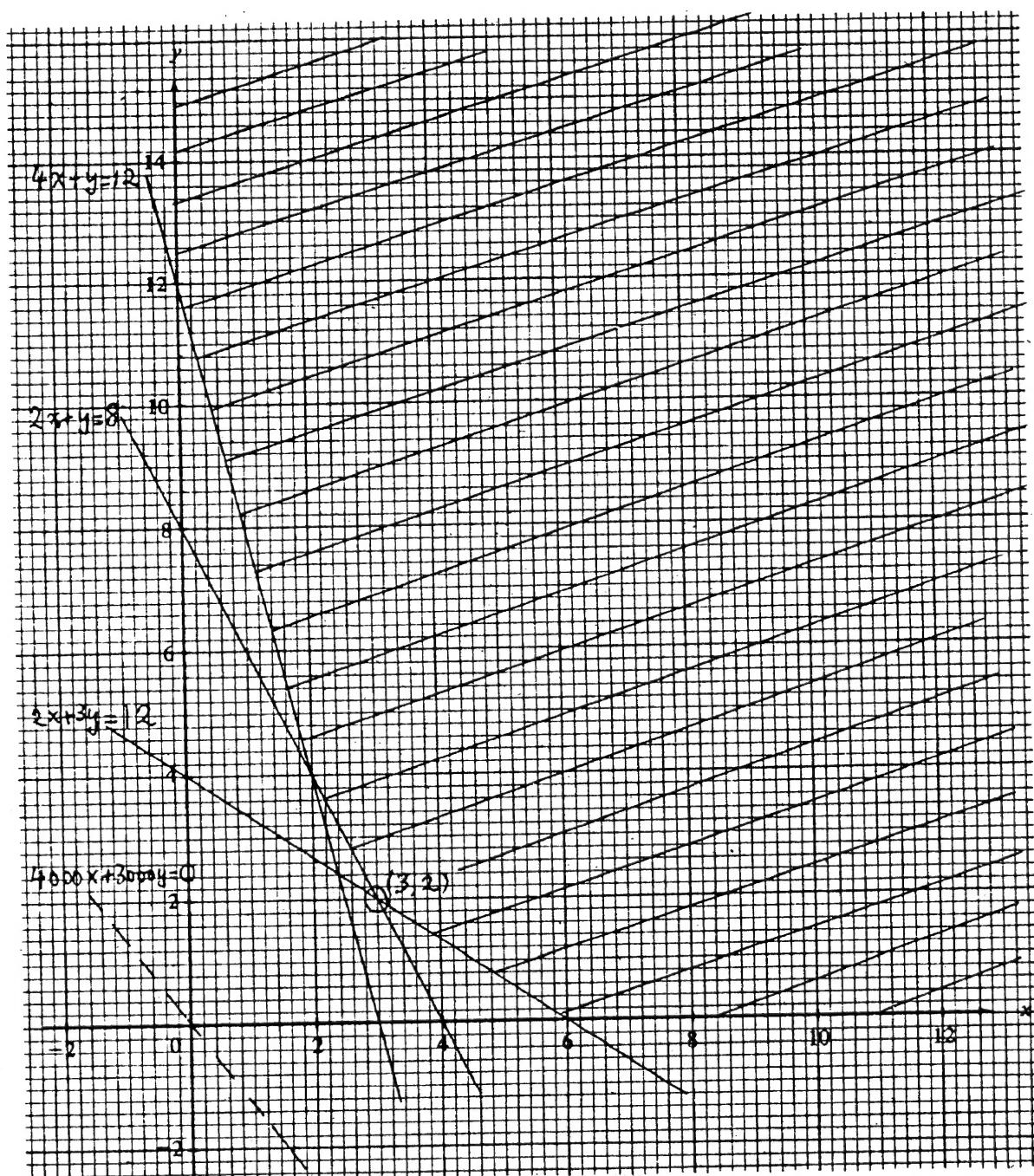


SOLUTION	MARKS	REMARKS
11. (a) Consider $\triangle ADE$. By the cosine rule $\begin{aligned} AE^2 &= AD^2 + DE^2 - 2AD \cdot DE \cos \angle ADE && \text{垂直} \\ &= 3^2 + 2^2 - 12 \cos 80^\circ (-= 10.91622) && \triangle ADE \text{ 之} \\ AE &= 3.304 \text{ cm (correct to 3 d.p.)} && \end{aligned}$	1M 1A 1A 3	correct use of formula
(b) Consider $\triangle ADE$ again. By the sine rule, $\begin{aligned} \frac{DE}{\sin \angle DAE} &= \frac{AE}{\sin \angle ADE} && \text{使用公式时之} \\ \sin \angle DAE &= \frac{DE \sin \angle ADE}{AE} && \text{公} \\ (-= \frac{1.9696}{3.304} = 0.59613) & && \end{aligned}$ $\angle DAE = 36.593^\circ \text{ (correct to 3 d.p.)}$ $\text{不} \angle DAE \text{ 不} 1A$	2M 1A 3	or cos rule Accept 36.593-36.594
(c) $DG = AD \sin \angle DAE$ $(= 3 \sin 36.593^\circ)$ $(= (3)(0.59613))$ $= 1.788 \text{ cm (correct to 3 d.p.)}$	1M 1A 2	or $\sin \angle DAE = \frac{DG}{AD}$ 不 1.788 cm 是
(d) $BD^2 = AB^2 + AD^2$ $BD = \sqrt{18}$ $= 4.243 \text{ cm (correct to 3 d.p.)}$	1M 1A 2	
(e) $\sin \angle DBG = \frac{DG}{BD}$ $(= \frac{1.788}{4.243} = 0.4214)$ $\therefore \angle DBG = 24.923^\circ \text{ (correct to 3 d.p.)}$	1M 1A 2	Accept 24.920-24.940



SOLUTION	MARKS	REMARKS
12. (a) Given that $x \geq 0$ $y \geq 0$ $4000x + 6000y \geq 24000$ Considering Products B and C, <u>$20000x + 5000y \geq 60000$</u> <u>$6000x + 3000y \geq 24000$</u>	1A 1A 2	Withhold 1A if '=' missing
(b) The constraints in (a) can be written as $x \geq 0$ $y \geq 0$ $2x + 3y \geq 12$ $4x + y \geq 12$ $2x + y \geq 8$ The lines corresponding to the last 3 inequalities are shown on the graph paper.	1A+1A +1A	+1 unit at x, y axes
Shading the correct region. Shading the correct region. The cost is least when $x = 3, y = 2$	3A 6	-1 if shading not complete. -2 if only arrows used
(c) Cost of materials used = $4000x + 3000y$ (dollars) Drawing the line $4000x + 3000y = 0$ (or equivalent) The cost is least when $x = 3, y = 2$ and the least cost is 18 000 (dollars)	1A 1M 1A 1A	Candidates may also test all vertices of given region. Awarded only if region correct
Point (6, 0) (3, 2) (2, 4) (0, 12)	Cost 24 000 18 000 20 000 36 000	4

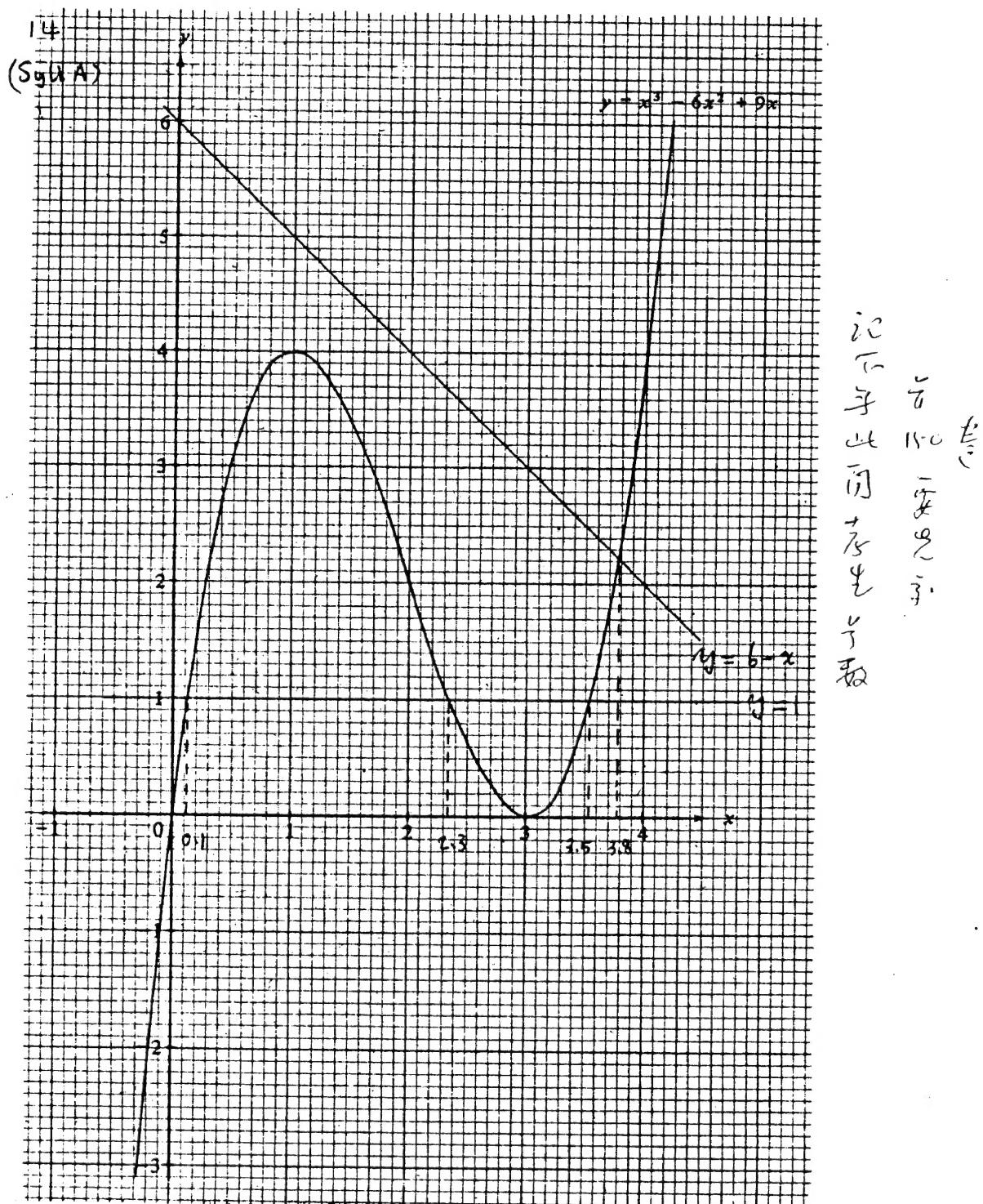
12.



SOLUTION	MARKS	REMARKS
13. (a) The probability that the black ball is not drawn = $\frac{5}{6}$ (or $1 - \frac{1}{6} = \frac{5}{6}$)	2A 2	Any value roundable to 0.83 P.P. if only answer is given. However, accept $P = 5/6$.
(2)		
(b) The probability that the black ball is drawn from P to Q in the 1st draw = $\frac{1}{6}$	1A	
(4) After that, the probability that the black ball is not drawn from Q to R in the 2nd draw = $\frac{4}{5}$	1A	
∴ the probability that the black ball is in Q		
$= \frac{1}{6} \times \frac{4}{5}$	$\frac{1}{6} + \frac{4}{5} \quad 1+1$	
$= \frac{2}{30} \quad (= \frac{4}{30})$	$\frac{1}{6} \times \frac{4}{5} \times \quad 1+1$	2A 4
(c) The probability that the black ball is drawn from Q to R = $\frac{1}{5}$	1A	Alternatively: $1 - \frac{5}{6} - \frac{2}{15}$ 2M
(1) ∴ the probability that the black ball is in R		
$= \frac{1}{6} \times \frac{1}{5}$	1A	
$= \frac{1}{30} \quad \therefore 0.03 \quad \frac{1}{6} \quad 1A$	1A 3	
(d) The probability that a white ball is drawn from P to Q in the 1st draw = $\frac{3}{6} \quad (= \frac{1}{2})$	1A	若不考虑丙球 概率有变
(3) After that, the probability that a white ball is drawn from Q to R in the 2nd draw = $\frac{1}{5}$	1A	若考虑丙球 概率不变 完全独立
∴ the probability that all balls in R are white = $\frac{1}{2} \times \frac{1}{5} \quad 1+1$	$1 - (\frac{3}{6} \times \frac{1}{5} + \frac{3}{6} \times \frac{4}{5})$	(PP-1)
$= \frac{1}{10} \quad \therefore 0.1 \quad \frac{1}{2} + \frac{3}{30}$	1A 3	

SOLUTION		MARKS	REMARKS								
(Syllabus A)											
14. (a) (i) $x^3 - 6x^2 + 9x - 1 = 0$											
(3) $x^3 - 6x^2 + 9x = 1$		1M									
Drawing the line $y = 1$, the roots of the given equation were found to be 0.1, 2.3 and 3.5 (correct to 1 d.p.).		1A+1A	1 mark for 2 correct answers								
(ii) $x^3 - 6x^2 + 10x - 6 = 0$											
(3) $x^3 - 6x^2 + 9x = 6 - x \dots$		1M	for correct L.S.								
Drawing the line $y = 6 - x$, the root was found to be 3.8 (correct to 1 d.p.)		1A	for graph, ±/unit at (3,3), (4,2)								
		1A									
		6									
(b)	<table border="1"> <thead> <tr> <th>x</th> <th>$x^3 - 6x^2 + 10x - 6$</th> </tr> </thead> <tbody> <tr> <td>3.76</td> <td>- (= -0.068)</td> </tr> <tr> <td>3.77</td> <td>+ (= 0.005)</td> </tr> <tr> <td>3.765</td> <td>- (= -0.031)</td> </tr> </tbody> </table>	x	$x^3 - 6x^2 + 10x - 6$	3.76	- (= -0.068)	3.77	+ (= 0.005)	3.765	- (= -0.031)		
x	$x^3 - 6x^2 + 10x - 6$										
3.76	- (= -0.068)										
3.77	+ (= 0.005)										
3.765	- (= -0.031)										
(3)											
		1M	Change of sign, -ve for 3.765-3.769								
		1A	May use graphical method								
		1A									
		3									
(c) Consider $x^3 - 6x^2 + 9x = k \leftarrow$	1M										
From the graph, if $0 < k < 4$,		1A+1A	-1 for ' $<$ ' if otherwise correct.								
the line $y = k$ meets the curve $y = x^3 - 6x^2 + 9x$ at three distinct points.			may omit								
$\therefore x^3 - 6x^2 + 9x - k = 0$ has three distinct roots.		3									

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SOLUTION	MARKS	REMARKS
(Syllabus B)		
14. (a) Since $y \propto x$ and $z \propto \frac{1}{x}$, $y = k_1 x$ and $z = \frac{k_2}{x}$ (for some real k_1, k_2). $\therefore p = k_1 x + \frac{k_2}{x}$ Putting $x = 2, p = 7$, (or $x = 3, p = 8$) $7 = 2k_1 + \frac{k_2}{2}$ i.e. $4k_1 + k_2 = 14$ Putting $x = 3, p = 8$. $8 = 3k_1 + \frac{k_2}{3} \dots \dots \dots$ or $9k_1 + k_2 = 24$ Solving these two equations, $5k_1 = 10$ $k_1 = 2$ $k_2 = 6$ $\therefore p = 2x + \frac{6}{x}$ When $x = 4, p = 2(4) + \frac{6}{4}$ $= \frac{19}{2} \dots \dots \dots$	1A+1A 1M 1A 1A	Accept $y = kx, z = \frac{k}{x}$
(b) $2x + \frac{6}{x} < 13$ $2x^2 - 13x + 6 < 0$ (as $x > 0$) $(2x - 1)(x - 6) < 0$ $\therefore \frac{1}{2} < x < 6$	1M 1A 2A 4	-1 for ' \leq '